LESSON 8.1

Relations and Functions

**BEFORE**

You graphed ordered pairs.

**Now**

You’ll use graphs to represent relations and functions.

**WHY?**

So you can show the growth of a bird over time, as in Ex. 26.

**Alligators** The table below shows the ages and lengths of five alligators.

<table>
<thead>
<tr>
<th>Age (years), x</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (in.), y</td>
<td>32</td>
<td>59</td>
<td>65</td>
<td>69</td>
<td>96</td>
</tr>
</tbody>
</table>

You can represent the relationship between age and length using the ordered pairs \((x, y)\):

\((2, 32), (4, 59), (5, 65), (5, 69), (7, 96)\)

The ordered pairs form a **relation**. A **relation** is a pairing of numbers in one set, called the **domain**, with numbers in another set, called the **range**. Each number in the domain is an **input**. Each number in the range is an **output**. For a relation represented by ordered pairs, the inputs are the \(x\)-coordinates and the outputs are the \(y\)-coordinates.

**Example 1** Identifying the Domain and Range

a. Identify the domain and range of the relation given above.

b. Identify the domain and range of the relation represented by the table below, which shows one alligator’s length at different ages.

<table>
<thead>
<tr>
<th>Age (years), x</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (in.), y</td>
<td>23</td>
<td>36</td>
<td>47</td>
<td>61</td>
<td>73</td>
</tr>
</tbody>
</table>

**Solution**

a. The domain of the relation is the set of all inputs, or \(x\)-coordinates. The range is the set of all outputs, or \(y\)-coordinates.

**Domain:** \(2, 4, 5, 7\)  
**Range:** \(32, 59, 65, 69, 96\)

b. The relation consists of the ordered pairs \((1, 23), (2, 36), (3, 47), (4, 61),\) and \((5, 73)\). The domain and range are shown below.

**Domain:** \(1, 2, 3, 4, 5\)  
**Range:** \(23, 36, 47, 61, 73\)

**Checkpoint**

Identify the domain and range of the relation.

1. \((0, 1), (2, 4), (3, 7), (5, 4)\)

2. \((-1, 2), (-3, -1), (6, 0), (-1, 4)\)
Representing Relations In addition to using ordered pairs or a table to represent a relation, you can also use a graph or a mapping diagram.

Example 2 Representing a Relation

Represent the relation \((-1, 1), (2, 0), (3, 1), (3, 2), (4, 5)\) as indicated.

a. A graph

b. A mapping diagram

Solution

a. Graph the ordered pairs as points in a coordinate plane.

b. List the inputs and the outputs in order. Draw arrows from the inputs to their outputs.

Functions A relation is a function if for each input there is exactly one output. In this case, the output is a function of the input.

Example 3 Identifying Functions

Tell whether the relation is a function.

a. The relation at the top of page 385, consisting of the ordered pairs (age, length) for five different alligators:

\[(2, 32), (4, 59), (5, 65), (5, 69), (7, 96)\]

b. The relation in part (b) of Example 1, consisting of the ordered pairs (age, length) for one alligator at different times:

\[(1, 23), (2, 36), (3, 47), (4, 61), (5, 73)\]

Solution

a. The relation is not a function because the input 5 is paired with two outputs, 65 and 69. This makes sense, as two alligators of the same age do not necessarily have the same length.

b. The relation is a function because every input is paired with exactly one output. This makes sense, as a single alligator can have only one length at a given point in time.

Checkpoint

Represent the relation as a graph and as a mapping diagram. Then tell whether the relation is a function. Explain your reasoning.

3. \[(0, 3), (1, 2), (2, -1), (4, 4), (5, 4)\]

4. \[(-2, -1), (0, 2), (2, 3), (-2, -4)\]
**Vertical Line Test** When a relation is represented by a graph, you can use the *vertical line test* to tell whether the relation is a function. The *vertical line test* says that if you can find a vertical line passing through more than one point of the graph, then the relation is not a function. Otherwise, the relation is a function.

### Example 4 Using the Vertical Line Test

**a.** In the graph below, no vertical line passes through more than one point. So, the relation represented by the graph is a function.

![Graph](image)

**b.** In the graph below, the vertical line shown passes through two points. So, the relation represented by the graph is not a function.

![Graph](image)

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### Guided Practice

#### Vocabulary Check

1. Copy and complete: A relation is a(n) _?_ if for each input there is exactly one output.

2. Draw a mapping diagram that represents a relation with domain \(-1, 0, 2\) and range \(1, 4\). Is only one answer possible? Explain.

#### Skill Check

**Identify the domain and range of the relation.**

3. \((0, 0), (1, 2), (2, 4), (3, 6), (4, 8)\)

4. \((2, 5), (-5, 2), (1, 5), (2, -3), (7, 5)\)

**Represent the relation as a graph and as a mapping diagram. Then tell whether the relation is a function. Explain your reasoning.**

5. \((1, 2), (1, 5), (2, 4), (3, 3), (4, 1)\)

6. \((-4, 2), (2, 3), (4, 8), (0, 3), (-2, 2)\)

7. **Error Analysis** Describe and correct the error in the given statement.

The relation \((1, -5), (2, -5), (3, 6), (4, 11)\) is not a function because the inputs 1 and 2 are both paired with the output \(-5\).
Identify the domain and range of the relation.

8. \((-2, 5), (-1, 2), (0, 4), (1, -9)\)
9. \((7, 3), (7, 6), (7, 9), (3, 3), (3, 6)\)

10. \[
\begin{array}{c|cccc}
 x & 4 & 2 & -3 & 4 \\
 y & 0 & -1 & 0 & -4 \\
\end{array}
\]

11. \[
\begin{array}{c|cccc}
 x & 1.5 & 1.5 & 2.8 & 2.8 \\
 y & 4.3 & 6.5 & 4.3 & 3.9 \\
\end{array}
\]

12. Copy and complete using always, sometimes, or never: A relation is _?_ a function.

Represent the relation as a graph and as a mapping diagram. Then tell whether the relation is a function. Explain your reasoning.

13. \((1, 2), (2, 1), (3, 0), (3, 4), (4, 3)\)
14. \((0, 4), (2, 0), (6, -4), (-4, 2), (8, 0)\)

15. \[
\begin{array}{c|cccc}
 x & -2 & -1 & 0 & 1 \\
 y & -3 & -3 & -3 & -3 \\
\end{array}
\]

16. \[
\begin{array}{c|cccc}
 x & 0 & -5 & -10 & 5 \\
 y & 15 & -10 & -10 & 5 \\
\end{array}
\]

17. **Height** The height of a person is measured every year from the age of 1 year to the age of 50 years.

a. Do the ordered pairs (age, height) represent a function? Explain.

b. **Critical Thinking** Would you expect the ordered pairs (height, age) to represent a function? Why or why not?

Tell whether the relation represented by the graph is a function.

18. ![Graph 1]
19. ![Graph 2]
20. ![Graph 3]

21. **Basketball** The table shows the numbers of games played and points scored by each starting player on the New Jersey Nets basketball team during the team’s 2001–2002 regular season.

<table>
<thead>
<tr>
<th>Player</th>
<th>Games played, x</th>
<th>Points scored, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Todd MacCulloch</td>
<td>62</td>
<td>604</td>
</tr>
<tr>
<td>Kenyon Martin</td>
<td>73</td>
<td>1086</td>
</tr>
<tr>
<td>Keith Van Horn</td>
<td>81</td>
<td>1199</td>
</tr>
<tr>
<td>Kerry Kittles</td>
<td>82</td>
<td>1102</td>
</tr>
<tr>
<td>Jason Kidd</td>
<td>82</td>
<td>1208</td>
</tr>
</tbody>
</table>

a. Identify the domain and range of the relation given by the ordered pairs \((x, y)\).

b. Draw a mapping diagram for the relation.

c. Is the relation a function? Explain.
22. **Hurricanes** In 1995, a total of 32 regular weather advisories were issued during the storm that became Hurricane Opal. The graph shows the wind speed inside Opal at the time of each advisory.

![Wind Speeds During Hurricane Opal](image)


b. **Estimation** An ocean storm is considered a hurricane if its wind speed is at least 74 miles per hour. For which advisories did Opal qualify as a hurricane?

23. **Writing** Suppose a relation is represented as a set of ordered pairs and as a mapping diagram. Which representation more clearly shows whether or not the relation is a function? Explain.

24. **Extended Problem Solving** A skydiver uses an altimeter to track altitude so that he or she knows when to open the parachute. The altimeter determines altitude by measuring changes in atmospheric pressure. The graph below shows how pressure varies with altitude as a skydiver falls from 12,000 feet to ground level. (The elevation of the ground is assumed to be 0 feet with respect to sea level.)

![Pressure Variation with Altitude](image)

a. As a skydiver falls, does the atmospheric pressure increase or decrease? Does the reading on the skydiver's altimeter increase or decrease?

b. **Writing** Describe the domain and range of the relation represented by the graph.

c. Is the relation a function? Explain.

d. **Interpret and Apply** Some altimeters can sound an alarm warning a skydiver to open the parachute when the altitude falls to a certain level. If the alarm is set to go off at an altitude of 3000 feet, approximately what atmospheric pressure will trigger the alarm?
25. **Challenge** To form the *inverse* of a relation represented by a set of ordered pairs, you switch the coordinates of each ordered pair. For example, the inverse of the relation (1, 2), (3, 4), (5, 6) is (2, 1), (4, 3), (6, 5). Give an example of a relation that is a function, but whose inverse is *not* a function.

26. **Birds** The brown-headed cowbird does not raise its own offspring. It lays eggs in the nests of other bird species, which then hatch the eggs and raise the young. A scientist investigated whether the growth of a young cowbird is affected by the species of bird that raises it. The scientist’s results for two bird species are shown below.

<table>
<thead>
<tr>
<th>Cowbird Raised by Red-Eyed Vireo</th>
<th>Cowbird Raised by Blue-Gray Gnatcatcher</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cowbird age (days)</td>
<td>Cowbird age (days)</td>
</tr>
<tr>
<td>0.0</td>
<td>0</td>
</tr>
<tr>
<td>2.0</td>
<td>2</td>
</tr>
<tr>
<td>4.0</td>
<td>4</td>
</tr>
<tr>
<td>6.0</td>
<td>6</td>
</tr>
<tr>
<td>8.0</td>
<td>8</td>
</tr>
<tr>
<td>10.0</td>
<td>10</td>
</tr>
<tr>
<td>12.0</td>
<td>12</td>
</tr>
<tr>
<td>14.0</td>
<td>14</td>
</tr>
<tr>
<td>Cowbird mass (grams)</td>
<td>Cowbird mass (grams)</td>
</tr>
<tr>
<td>1.7</td>
<td>2.2</td>
</tr>
<tr>
<td>4.5</td>
<td>5.2</td>
</tr>
<tr>
<td>10.7</td>
<td>10.1</td>
</tr>
<tr>
<td>20.0</td>
<td>15.5</td>
</tr>
<tr>
<td>28.3</td>
<td>19.3</td>
</tr>
<tr>
<td>33.0</td>
<td>21.2</td>
</tr>
<tr>
<td>35.0</td>
<td>22.0</td>
</tr>
<tr>
<td>35.7</td>
<td>22.3</td>
</tr>
</tbody>
</table>

- a. For each table, draw a graph for the relation given by the ordered pairs (age, mass). Draw both graphs in the same coordinate plane, and use a different color for each graph.

- b. **Interpret** Compare the graphs from part (a). How is a cowbird’s growth when raised by a red-eyed vireo like its growth when raised by a blue-gray gnatcatcher? How is its growth different?

**Mixed Review**

Evaluate the expression when \( x = -5 \) and \( y = -7 \). (Lessons 1.5–1.7)

27. \( x + y \)  
28. \( y - x + 10 \)  
29. \( 2x^2y \)  
30. \( 3x - 4y \)

Tell whether the given value of the variable is a solution of the equation. (Lesson 2.4)

31. \( x + 11 = 3; x = -8 \)  
32. \( -17 - a = -23; a = -6 \)  
33. \( -6m = -84; m = 13 \)  
34. \( \frac{-144}{u} = 12; u = -12 \)

For an account that earns simple annual interest, find the interest and the balance of the account. (Lesson 7.7)

35. \( P = 850, r = 3\% , t = 6 \text{ years} \)  
36. \( P = 4200, r = 5\% , t = 7.5 \text{ years} \)

**Standardized Test Practice**

37. **Extended Response** The table shows the amount charged for standard ground shipping by an online electronics store.

<table>
<thead>
<tr>
<th>Total cost of merchandise</th>
<th>Shipping cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>$.01–$25.00</td>
<td>$5.95</td>
</tr>
<tr>
<td>$25.01–$50.00</td>
<td>$7.95</td>
</tr>
<tr>
<td>$50.01–$75.00</td>
<td>$9.95</td>
</tr>
<tr>
<td>$75.01–$100.00</td>
<td>$11.95</td>
</tr>
<tr>
<td>Over $100.00</td>
<td>$13.95</td>
</tr>
</tbody>
</table>
