Lesson 8.6
Writing Linear Equations

**Vocabulary**
best-fitting line, p. 421

**BEFORE**
You graphed linear equations.

**Now**
You’ll write linear equations.

**WHY?**
So you can describe the area of glaciers, as in Ex. 26.

**Bamboo** Bamboo is one of the fastest-growing plants on Earth. It can grow up to 4 feet in one day! In Example 4, you’ll see how to write a linear equation that describes the growth of a bamboo plant.

You can write a linear equation in slope-intercept form, \( y = mx + b \), if you know the slope \( m \) and the \( y \)-intercept \( b \) of the equation’s graph.

**Example 1**
**Writing an Equation Given the Slope and \( y \)-Intercept**

Write an equation of the line with a slope of 3 and a \( y \)-intercept of \(-7\).

\[ y = mx + b \]

Write general slope-intercept equation.

\[ y = 3x + (-7) \]

Substitute 3 for \( m \) and \(-7\) for \( b \).

\[ y = 3x - 7 \]

Simplify.

**Example 2**
**Writing an Equation of a Graph**

Write an equation of the line shown.

1. Find the slope \( m \) using the labeled points.

\[ m = \frac{2 - 3}{4 - 0} = \frac{-1}{4} = -\frac{1}{4} \]

2. Find the \( y \)-intercept \( b \). The line crosses the \( y \)-axis at \((0, 3)\), so \( b = 3 \).

3. Write an equation of the form \( y = mx + b \).

\[ y = -\frac{1}{4}x + 3 \]

**Checkpoint**

1. Write an equation of the line with a slope of 1 and a \( y \)-intercept of 5.

2. Write an equation of the line through the points \((-2, 6)\) and \((0, -4)\).
Example 3  Writing Equations of Parallel or Perpendicular Lines

a. Write an equation of the line that is parallel to the line \( y = 4x - 8 \) and passes through the point \((0, 2)\).

b. Write an equation of the line that is perpendicular to the line \( y = -5x + 1 \) and passes through the point \((0, -9)\).

Solution

a. The slope of the given line is 4, so the slope of the parallel line is also 4. The parallel line passes through \((0, 2)\), so its \( y \)-intercept is 2.

Answer An equation of the line is \( y = 4x + 2 \).

b. Because the slope of the given line is \(-5\), the slope of the perpendicular line is the negative reciprocal of \(-5\), or \( \frac{1}{5} \). The perpendicular line passes through \((0, -9)\), so its \( y \)-intercept is \(-9\).

Answer An equation of the line is \( y = \frac{1}{5}x + (-9) \), or \( y = \frac{1}{5}x - 9 \).

Example 4  Writing an Equation from a Table

The table shows a bamboo plant’s growth over 8 hours. Show that the table represents a linear function. Write an equation for the function.

<table>
<thead>
<tr>
<th>Time (h), ( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (in.), ( y )</td>
<td>6</td>
<td>10</td>
<td>14</td>
<td>18</td>
<td>22</td>
</tr>
</tbody>
</table>

Solution

1. Make a scatter plot. The points lie on a nonvertical line, so the table represents a linear function.

2. Find the slope \( m \) using any two points on the line, such as \((0, 6)\) and \((2, 10)\).

\[
m = \frac{10 - 6}{2 - 0} = \frac{4}{2} = 2
\]

3. Find the \( y \)-intercept \( b \). The line intersects the \( y \)-axis at \((0, 6)\), so \( b = 6 \).

4. Write the equation \( y = mx + b \).

\[
y = 2x + 6
\]

In the Real World

Bamboo  Bamboo is a rapidly renewable building material compared to trees such as oak. Bamboo takes about 5 years to grow, while oak takes about 120 years. How many bamboo forests can be grown and harvested in the time it takes to grow one oak forest?

Checkpoint

3. Which representation of a function more clearly shows whether or not the function is linear: a table of values or a graph? Explain.
Best-Fitting Lines  In Example 4, the points in the scatter plot lie exactly on a line. Often, however, there is no single line that passes through all the points in a data set. In such cases, you can find the best-fitting line, which is the line that lies as close as possible to the data points.

The following example uses a graphical method to approximate the equation of a best-fitting line. In the activity on page 425, you’ll use a graphing calculator to find a better approximation of this line.

Example 5  Approximating a Best-Fitting Line

Medicine  The table shows the number of female physicians in the United States for the years 1992–1999.

<table>
<thead>
<tr>
<th>Years since 1992, x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female physicians (in thousands), y</td>
<td>110</td>
<td>117</td>
<td>125</td>
<td>140</td>
<td>148</td>
<td>158</td>
<td>168</td>
<td>177</td>
</tr>
</tbody>
</table>

a. Approximate the equation of the best-fitting line for the data.
b. Predict the number of female physicians in 2005.

Solution

a. First, make a scatter plot of the data pairs.

Next, draw the line that appears to best fit the data points. There should be about the same number of points above the line as below it. The line does not have to pass through any of the data points.

Finally, write an equation of the line. To find the slope, estimate the coordinates of two points on the line, such as (0, 108) and (7, 177).

\[ m = \frac{177 - 108}{7 - 0} = \frac{69}{7} = 9.86 \]

The line intersects the y-axis at (0, 108), so the y-intercept is 108.

Answer  An approximate equation of the best-fitting line is \( y = 9.86x + 108 \).

b. Note that 2005 − 1992 = 13, so 2005 is 13 years after 1992. Calculate \( y \) when \( x = 13 \) using the equation from part (a).

\[ y = 9.86(13) + 108 \]

Answer  In 2005, there will be about 236,000 female physicians in the United States.
Guided Practice

**Vocabulary Check**
1. Copy and complete: The line that lies as close as possible to the data points in a scatter plot is called the ___.
2. Describe the steps you would use to write an equation of the line through the points (−2, 3) and (0, 9).

**Skill Check**

**Write an equation of the line through the given points.**
3. (0, 8), (1, 9)  4. (−2, 13), (0, 1)  5. (0, −5), (3, −3)
6. Write an equation of the line that is perpendicular to the line $y = 2x − 11$ and passes through the point (0, −7).

**Guided Problem Solving**

**Clams** The table shows the dimensions of seven butter clams. What is the approximate length of a butter clam that is 85 millimeters wide?

<table>
<thead>
<tr>
<th>Width (mm), $x$</th>
<th>13</th>
<th>21</th>
<th>30</th>
<th>39</th>
<th>50</th>
<th>60</th>
<th>71</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (mm), $y$</td>
<td>17</td>
<td>28</td>
<td>40</td>
<td>52</td>
<td>62</td>
<td>77</td>
<td>91</td>
</tr>
</tbody>
</table>

1. Make a scatter plot of the data pairs. Draw the line that appears to best fit the data points.
2. Write an equation of your line.
3. Use your equation to predict, to the nearest millimeter, the length of a butter clam that is 85 millimeters wide.

Practice and Problem Solving

**Write an equation of the line with the given slope and $y$-intercept.**
8. slope = −3; $y$-intercept = 5  9. slope = 4; $y$-intercept = 10
10. slope = 13; $y$-intercept = −8  11. slope = −1; $y$-intercept = −20

**Write an equation of the line.**
12. [Diagram showing a line with points (0, 1) and (1, 4)]
13. [Diagram showing a line with points (0, 2) and (3, 0)]
14. [Diagram showing a line with points (0, 3) and (2, −1)]

**Write an equation of the line through the given points.**
15. (0, 9), (3, 15)  16. (0, −6), (8, −16)  17. (−2, −11), (0, −11)
Write an equation of the line that is parallel to the given line and passes through the given point.

18. \(y = 2x + 1; (0, 4)\)  
19. \(y = -x - 3; (0, 7)\)  
20. \(y = -8x + 9; (0, -2)\)

Write an equation of the line that is perpendicular to the given line and passes through the given point.

21. \(y = 3x + 4; (0, 6)\)  
22. \(y = x - 7; (0, -5)\)  
23. \(y = -\frac{1}{4}x + 3; (0, 1)\)

Show that the table represents a linear function. Then write an equation for the function.

24. 
\[
\begin{array}{c|c|c|c|c|c}
 x & -2 & -1 & 0 & 1 & 2 \\
 y & -5 & -2 & 1 & 4 & 7 \\
\end{array}
\]

25. 
\[
\begin{array}{c|c|c|c|c|c}
 x & 0 & 2 & 4 & 6 & 8 \\
 y & -3 & -2 & -1 & 0 & 1 \\
\end{array}
\]

26. **Extended Problem Solving** Since 1912, scientists have created five maps of the glaciers on top of Mount Kilimanjaro in Africa. The maps indicate that the glaciers are shrinking, as shown by the table.

<table>
<thead>
<tr>
<th>Map number</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year map was made</td>
<td>1912</td>
<td>1953</td>
<td>1976</td>
<td>1989</td>
<td>2000</td>
</tr>
<tr>
<td>Area of glaciers (km²)</td>
<td>12.1</td>
<td>6.7</td>
<td>4.2</td>
<td>3.3</td>
<td>2.2</td>
</tr>
</tbody>
</table>

**a. Graph** Let \(x\) be the number of years since 1912. Let \(y\) be the area of the glaciers (in square kilometers). Make a scatter plot of the data pairs \((x, y)\). Draw the line that appears to best fit the data points.

**b. Represent** Write an equation of your line.

**c. Predict** Estimate the year when the glaciers will disappear.

Two variables \(x\) and \(y\) show **direct variation** if \(y = kx\) for some nonzero number \(k\). In Exercises 27–30, write a direct variation equation that has the given ordered pair as a solution.

**Example** **Writing a Direct Variation Equation**

Write a direct variation equation that has \((4, 20)\) as a solution.

\[
\begin{align*}
y &= kx \\
20 &= k(4) \\
5 &= k \\
\text{Answer} & \quad \text{A direct variation equation is } y = 5x.
\end{align*}
\]

27. \((5, 15)\)  
28. \((-3, 21)\)  
29. \((-8, -4)\)  
30. \((12, -16)\)

**Sales** Lisa and John work in different department stores. Lisa earns a salary of $18,000 per year plus a 2% commission on her sales. John receives no salary but earns a 6% commission on his sales. For each person, tell whether annual sales and annual earnings show direct variation. Justify your answers mathematically.
32. **Physics** The table below gives the length of a spring when different masses are suspended from it.

<table>
<thead>
<tr>
<th>Mass (g), x</th>
<th>0</th>
<th>50</th>
<th>100</th>
<th>150</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (mm), y</td>
<td>80</td>
<td>110</td>
<td>140</td>
<td>170</td>
<td>200</td>
</tr>
</tbody>
</table>

a. Show that the table represents a linear function.

b. Write an equation for the function.

33. **Marathons** The table below shows the men’s winning times in the Boston Marathon for every tenth year from 1900 to 2000. In the table, \( x \) represents the number of years since 1900, and \( y \) represents the corresponding winning time (to the nearest minute).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
<th>90</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>160</td>
<td>149</td>
<td>150</td>
<td>155</td>
<td>148</td>
<td>153</td>
<td>141</td>
<td>131</td>
<td>132</td>
<td>128</td>
<td>130</td>
</tr>
</tbody>
</table>

a. Make a scatter plot of the data pairs \((x, y)\). Draw the line that appears to best fit the data points.

b. Write an equation of your line.

c. **Predict** Use your equation to predict, to the nearest minute, the men’s winning time in the Boston Marathon for the year 2010.

d. **Writing** Do you think your equation will accurately predict winning times far into the future? Explain your reasoning.

34. **Challenge** Write an equation of the line through \((2, -1)\) and \((6, 5)\). Describe the method you used to determine the equation.

35. Solve the equation. Check your solution. *(Lesson 3.3)*

\[
35. \quad 8x - 5 = 5x + 7 \\
36. \quad -7y + 4 = -y + 22 \\
37. \quad 4(m - 4) = 2m \\
38. \quad 6(1 - n) = -6n + 1
\]

36. **Write the fraction as a percent.** *(Lesson 7.3)*

\[
39. \quad \frac{7}{10} \\
40. \quad \frac{3}{8} \\
41. \quad \frac{5}{2} \\
42. \quad \frac{9}{5}
\]

37. **Identify the slope and \( y \)-intercept of the line with the given equation. Use the slope and \( y \)-intercept to graph the equation.** *(Lesson 8.5)*

\[
43. \quad y = 3x - 2 \\
44. \quad y = -x + 5 \\
45. \quad 3x + 2y = 0 \\
46. \quad x - 2y = -2
\]

38. **Multiple Choice** What is an equation of the line through the points \((0, 8)\) and \((2, 0)\)?

A. \( y = 4x + 2 \)  
B. \( y = 4x + 8 \)  
C. \( y = -4x + 2 \)  
D. \( y = -4x + 8 \)

39. **Multiple Choice** What is an equation of the line that is parallel to the line \( y = 5x + 3 \) and passes through the point \((0, -1)\)?

F. \( y = 5x - 1 \)  
G. \( y = -5x - 1 \)  
H. \( y = \frac{1}{5}x - 1 \)  
I. \( y = -\frac{1}{5}x - 1 \)