**Lesson 10.3**

**Areas of Parallelograms and Trapezoids**

**Vocabulary**
- base of a parallelogram, p. 521
- height of a parallelogram, p. 521
- bases of a trapezoid, p. 522
- height of a trapezoid, p. 522

**Before**
You classified polygons. You’ll find the areas of parallelograms and trapezoids. So you can compare the areas of two parking lots, as in Ex. 26.

The **base of a parallelogram** is the length of any one of its sides. The **height of a parallelogram** is the perpendicular distance between the side whose length is the base and the opposite side. The diagrams below show how to change a parallelogram into a rectangle with the same base, height, and area as the parallelogram.

Notice that the area of the rectangle above is the product of the base \( b \) and the height \( h \). The diagram suggests the formula below.

**Area of a Parallelogram**

**Words** The area \( A \) of a parallelogram is the product of the base \( b \) and the height \( h \).

**Algebra** \( A = bh \)

**Numbers** \( A = 8 \cdot 6 = 48 \text{ m}^2 \)

**Example 1** *Finding the Area of a Parallelogram*

The base of a parallelogram is 5 inches. The height is twice the base. Find the area of the parallelogram.

1. Find the height.
   
   \[ h = 2b \]
   
   \[ = 2(5) \]
   
   \[ = 10 \text{ in.} \]

2. Find the area.
   
   \[ A = bh \]
   
   \[ = 5(10) \]
   
   \[ = 50 \text{ in.}^2 \]

**Answer** The parallelogram has an area of 50 square inches.
### Trapezoids

The **bases of a trapezoid** are the lengths of its parallel sides. The **height of a trapezoid** is the perpendicular distance between the sides whose lengths are the bases. The diagram below shows how two congruent trapezoids with height \( h \) and bases \( b_1 \) and \( b_2 \) can be put together to form a parallelogram with base \( b_1 + b_2 \) and height \( h \).

![Trapezoid Diagram](image)

Notice the area of the parallelogram is twice the area of either trapezoid. This result suggests the formula below.

#### Area of a Trapezoid

**Words** The area \( A \) of a trapezoid is one half of the product of the sum of the bases, \( b_1 \) and \( b_2 \), and the height, \( h \).

**Algebra** \[ A = \frac{1}{2}(b_1 + b_2)h \]

**Numbers** \[ A = \frac{1}{2}(5 + 7)4 = 24 \text{ cm}^2 \]

#### Example 2

**Quilts** The diagram shows one of the trapezoids in a quilt design. Find the area of the trapezoid.

**Solution**

\[
A = \frac{1}{2}(b_1 + b_2)h \\
= \frac{1}{2}(4 + 9)2.5 \\
= 16.25
\]

**Answer** The trapezoid has an area of 16.25 square centimeters.

**Checkpoint**

Find the area of the parallelogram or trapezoid.

1. \( 5 \text{ ft} \) \( 6 \text{ ft} \)
2. \( 3 \text{ m} \) \( 8.5 \text{ m} \)
3. \( 22 \text{ in.} \) \( 16 \text{ in.} \) \( 13 \text{ in.} \)
The height of a trapezoid is 6 meters. One of its bases is 8 meters. The area of the trapezoid is 54 square meters. Find the other base.

Write formula for area of a trapezoid.

Substitute 54 for \( A \), 8 for \( b_1 \), and 6 for \( h \).

Multiply.

Distributive property

Subtract 24 from each side.

Divide each side by 3.

Answer The other base is 10 meters.

Example 3

Finding an Unknown Length

The height of a trapezoid is 6 meters. One of its bases is 8 meters. The area of the trapezoid is 54 square meters. Find the other base.

\[ A = \frac{1}{2} (b_1 + b_2)h \]

Write formula for area of a trapezoid.

Substitute 54 for \( A \), 8 for \( b_1 \), and 6 for \( h \).

\[ 54 = \frac{1}{2} (8 + b_2)6 \]

Multiply.

\[ 54 = 3(8 + b_2) \]

Distributive property

\[ 54 = 24 + 3b_2 \]

Subtract 24 from each side.

\[ 30 = 3b_2 \]

Divide each side by 3.

Answer The other base is 10 meters.

Example 4

Using Area of Trapezoids

Desk You are building an L-shaped desk for your room. The dimensions of the desktop are shown. Find the area of the desktop.

Solution

1. Divide the desktop into two trapezoids, A and B, as shown.

2. Find the sum of the areas of trapezoids A and B.

\[ \text{Area of trapezoid } A = \frac{1}{2} (b_1 + b_2)h \]

\[ = \frac{1}{2} (5 + 8)3 = \frac{39}{2} = 19 \frac{1}{2} \]

\[ \text{Area of trapezoid } B = \frac{1}{2} (b_1 + b_2)h \]

\[ = \frac{1}{2} (4 + 9)3 = \frac{39}{2} = 19 \frac{1}{2} \]

3. Add the areas.

\[ \text{Area of trapezoid } A + \text{Area of trapezoid } B = 19 \frac{1}{2} + 19 \frac{1}{2} = 39 \]

Answer The total area of the desktop is 39 square feet.

Checkpoint

4. One base of a trapezoid is 9 feet, and the height is 4 feet. The area of the trapezoid is 28 square feet. Find the other base.
### Guided Practice

**Vocabulary Check**

1. Sketch a trapezoid and label its bases and height. State the formula for finding its area.

2. The height of a parallelogram is 22 inches. The base is one half of the height. Find the area of the parallelogram.

**Skill Check**

1. Sketch a trapezoid and label its bases and height. State the formula for finding its area.

2. Find the area of the trapezoid.

   - **Exercise 3:**
     - Bases: 9 ft and 12 ft
     - Height: 16 ft
   - **Exercise 4:**
     - Bases: 35 in. and 70 in.
     - Height: 35 in.
   - **Exercise 5:**
     - Bases: 48 m and 62 m
     - Height: 88 m

3. Find the unknown base or height of the parallelogram.

   - **Exercise 6:**
     - Area: 40 in.², Base: 25 in., Height: ?
   - **Exercise 7:**
     - Area: 300 m², Base: ?, Height: 20 m

4. Find the unknown base or height of the trapezoid.

   - **Exercise 8:**
     - Area: 12 ft², Base 1: 2 ft, Base 2: ?, Height: 3 ft
   - **Exercise 9:**
     - Area: 240 m², Base 1: 16 m, Base 2: 8 m, Height: ?

10. **Track Uniform**

    You are sewing a red stripe on the front of a track uniform. As shown, the stripe is a parallelogram. What is the area of the stripe?

### Practice and Problem Solving

**Homework Help**

- **Example 1:** Exercises 11-13, 17
- **Example 2:** Exercises 14-16, 18
- **Example 3:** Exercises 19-24
- **Example 4:** Exercises 25, 26

**Online Resources**

- More Examples
- eTutorial Plus

1. Find the area of the parallelogram.

   - **Exercise 11:**
     - Bases: 5 in. and 14 in.
     - Height: 14 in.
   - **Exercise 12:**
     - Bases: 8 yd and 9.5 yd
     - Height: 8 yd
   - **Exercise 13:**
     - Bases: 8.3 mm and 11.5 mm
     - Height: 8.3 mm

2. Find the area of the trapezoid.

   - **Exercise 14:**
     - Bases: 14 ft and 12 ft
     - Height: 18 ft
   - **Exercise 15:**
     - Bases: 3.2 m and 3.6 m
     - Height: 7 m
   - **Exercise 16:**
     - Bases: 7.5 cm and 19 cm
     - Height: 10.2 cm
17. The base of a parallelogram is 10 meters. The height is one fourth of the base. Find the area of the parallelogram.

18. The height of a trapezoid is 2 feet. One of the bases is three times the height, and the other base is four times the height. Find the area of the trapezoid.

**Find the unknown measure of the parallelogram.**

19. \( A = 2025 \text{ m}^2 \)

20. \( A = 71.5 \text{ in.}^2 \)

21. \( A = 1 \text{ yd}^2 \)

**Find the unknown measure of the trapezoid.**

22. \( A = 192.5 \text{ cm}^2 \)

23. \( A = 1800 \text{ ft}^2 \)

24. \( A = 16.555 \text{ mm}^2 \)

25. **Aircraft Wings** A wing of each aircraft described has the shape of a trapezoid. Find the area of the wing.

   a. An F-18 wing has bases of 6 feet and 15 feet and height of 13 feet.
   
   b. A Boeing 747 wing has bases of 13.3 feet and 54.3 feet and height of 81.3 feet.

26. **Parking Lot** Two parking lots each have space for 5 cars, as shown in the diagrams below.

   a. Find the base of each figure formed by the 5 parking spaces.
   
   b. Find the area of each figure formed by the 5 parking spaces.
   
   c. **Compare** Which parking lot covers less area to park 5 cars?

27. In Exercises 27 and 28, plot the points in a coordinate plane. Connect the points so that they form a polygon. Identify the polygon and find its area.

   27. \((-2, -3), (-2, 0), (2, 3), (2, -4)\)
   
   28. \((-1, 3), (4, 3), (2, -1), (-3, -1)\)

29. **Writing** What happens to the area of a trapezoid if you double its height? if you double both its bases? if you double the height and both bases?
Find the area of the figure.

30.

31.

32. **Summer Camp** This summer at camp, you can stay in room A or room B with one roommate. Which room will give you and your roommate more space?

Room A

Room B

33. **Picture Frame** You have a 4 inch by 6 inch picture that you want to have framed. You want the frame to be 2 inches wide. A wooden frame can be made from four trapezoids, as shown. Find the areas of the bottom and side trapezoids. Then find the ratio of the area of the bottom trapezoid to the area of the side trapezoid.

34. **Challenge** You form a rhombus by putting two equilateral triangles with side length $2n$ together, as shown. Write an expression for the area of the rhombus in terms of $n$. Explain your reasoning.

**Mixed Review**

For an account that earns interest compounded annually, find the balance of the account. Round to the nearest cent. *(Lesson 7.7)*

35. $P = $1200, $r = 5\%$, $t = 3$ years  
36. $P = $8550, $r = 3.5\%$, $t = 20$ years

Approximate the square root to the nearest integer. *(Lesson 9.1)*

37. $\sqrt{40}$  
38. $\sqrt{587}$  
39. $\sqrt{10.2}$  
40. $\sqrt{0.725}$

41. Find the value of $x$ in the quadrilateral shown. *(Lesson 10.2)*

**Standardized Test Practice**

42. **Multiple Choice** The height of a parallelogram is 13.5 feet. The base is four times the height. What is the area of the parallelogram?

- A. 45.5625 ft$^2$  
- B. 54 ft$^2$  
- C. 182.25 ft$^2$  
- D. 729 ft$^2$

43. **Short Response** Is it possible for two parallelograms to have the same area but not be congruent? Explain why or why not.